# THE MATHEMATICAL ASSOCIATION

National Committee for Mathematical Contests

Further International Selection Test

Thursday, May 13, 1982

Time $3\frac{1}{2}$ hours

1. ABC is a triangle. The internal bisector of the angle A meets the circumcircle again at P. Q and R are similarly defined.

   Prove that $AP + BQ + CR > AB + BC + CA$.

2. The sequence $p_1, p_2, \ldots$ is defined as follows -

   $p_1 = 2$, and for $n \geq 2$, $p_n$ is the largest prime divisor of

   $p_1 p_2 p_3 \cdots p_{n-1} + 1$.

   Prove that 5 is not a member of this sequence.

3. Find the largest positive integer n for which the equation

   $$ax + (a+1)y + (a+2)z = n$$

   is not solvable in positive integers x,y,z, where a is a given odd positive integer.

4. $P_1(x_1, y_1)$, $P_2(x_2, y_2)$ are two points on that part of the curve $x^n - ay^n = b$ for which $x > 0$, $y > 0$. Here a and b are positive constants and n an integer > 1. Prove that if $y_1 < y_2$, and $\Delta$ is the area of triangle $OP_1P_2$, then

   $$by_2 > 2ny_1^{n-1} a^{1-\frac{1}{n}} \Delta .$$

5. Given that k is a fixed non-negative integer and that the polynomial P(x) satisfies the relation

   $P(2x) = 2^{k-1}(P(x) + P(x+\frac{1}{2}))$, prove that

   $P(3x) = 3^{k-1}(P(x) + P(x+\frac{1}{3}) + P(x+\frac{2}{3}))$.